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Feedback Theory

An introduction to quasi-static feedback analysis

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This paper presents an intuitive approach to analyzing quasi-static feedback circuits. This approach involves decomposing feedback amplifiers into flow-graph diagrams, rather than applying 2-port theory, which is shown to be useless for analyzing many feedback amplifiers. Using the feedback analysis, we derive methods for determining the gain, input resistance, and output resistance of a feedback amplifier, then work out various examples for applying feedback theory, ranging in complexity from a simple resistive divider to a complex multiple-loop feedback amplifier.

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1. Introduction

Feedback theory is one of the more difficult facets of circuit analysis for which to develop an intuitive understanding. A firm grasp of feedback analysis requires a grounding in circuit analysis techniques as a foundation, to include Kirchhoff's voltage and current laws, current and voltage divider equations, superposition, Thévenin and Norton equivalent circuits, and transistor gain equations, to name a few. Feedback analysis is usually applied to circuits which contain active components, such as transistors or operational amplifiers, although we will show that feedback analysis can be applied to purely passive resistive networks as well. Feedback theory provides insights into circuit behavior which are not as visible using lower-level circuit analysis.

The main difficulty when it comes to applying feedback theory to a circuit is to understand how to decompose a circuit using flow-graph diagrams. This involves identifying current and voltage nodes in a circuit and determining how those nodes interact. Another challenge is to understand how feedback has an effect in the input and output resistance of an amplifier. In this paper, we will begin by going over how to construct and understand flow-graph diagrams, and learn how they are useful for understanding circuits from a feedback perspective. We will work through several examples of feedback circuits starting with resistive dividers and work towards harder multiple-feedback examples. By doing so, we will attempt to develop an intuitive understanding of feedback circuits. We will analyze only quasi-static circuits, meaning that we will ignore all dynamic circuit elements such as inductors and capacitors.

A study of 2-port network theory is useful for gaining some of the intuition needed behind finding input and output resistances, and for using the amplifiers as discrete units. Most text books use 2-port network theory quite heavily, which has a tendency of making feedback analysis a bit more complicated than necessary. In this paper, we will use mainly flow-graph diagrams in our feedback and touch only minimally on 2-port network theory.

2. Flow-graph Diagrams

To express a feedback system, we will use what are known as flow-graph diagrams. The flow-graph diagram is useful, since it provides insight into how various circuit quantities relate with each other in a system. A flow-graph diagram for a typical single-path feedback amplifier is shown below.

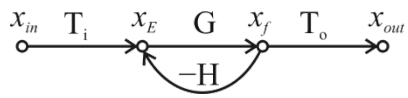

Figure 2.1. Flow-graph for typical single-loop feedback amplifier

The flow-graph diagram can also be expressed as a block diagram, which is typically used in signal processing diagrams. An equivalent block diagram is shown in Figure 2.2.

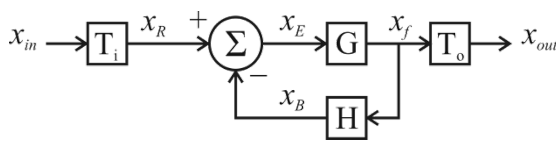

Figure 2.2. Block diagram for typical single-loop feedback amplifier

The block diagram, although much more commonly used, is not as easy to draw for more complicated systems as the flow-graph diagram. We will therefore use the flow-graph feedback representation for the remainder of this paper.

In Figure 2.1, we see a flow-graph comprised of **nodes** (circles) connected to each other through **paths** (arrows). Each path begins at an **origin node** and ends at a **destination node** and is labeled with the **path gain**. A path is a **feedback path** if it has a destination node closer to the input node than the origin node, which includes the H path in our example. The nodes are labeled by the **node quantities**, which begin with the letter x and represent either a voltage or current in the amplifier circuit. The paths represent how each node affects the other nodes in the system. The node x_E for instance is affected by nodes x_{in} by the path gain $\mathrm{T_i}$ and x_f by the path gain -H. For each node, we can derive an algebraic equation called the **node path equation**, which is the sum of the products of the node quantities and path gains which lead to each node. The node path equations for the system in Figure 2.1 are as follows:

$$x_E = \mathrm{T_i}\, x_{in} - \mathrm{H}\, x_f \qquad \textbf{(2.1)}$$

$$x_f = \mathrm{G}\, x_E \qquad \textbf{(2.2)}$$

$$v_{out} = \mathrm{T_o}\, x_f \qquad \textbf{(2.3)}$$

By solving the system of equations given by the node path equations, we can solve for all the **node gain equations** in our system. The node gain equations state the ratio of any node quantity to the input node quantity.

$$\frac{x_E}{x_{in}} = \frac{\mathrm{T_i}}{1+\mathrm{G}\,\mathrm{H}} \qquad \textbf{(2.4)}$$

$$\frac{x_f}{x_{in}} = \frac{\mathrm{T_i\,G}}{1+\mathrm{G\,H}} \qquad \textbf{(2.5)}$$

$$\frac{x_{out}}{x_{in}} = \frac{\mathrm{T_i\,T_o\,G}}{1+\mathrm{G\,H}} \qquad \textbf{(2.6)}$$

The node gain equations all have a denominator of 1 + G H. The product G H is called the **loop gain**. The **closed-loop gain** is the total gain for the system, which is given in equation (2.6).The numerator of the closed-loop gain, T_i T_o G, is the **open-loop gain** of the system, or the gain of the system after nulling the feedback paths.

In the block diagram representation of the feedback system, two extra signals appear: x_R and x_B. These signals are realized in the following flow-graph diagram. These two nodes do not usually have realizable counterparts in electrical circuits, although they will become helpful later for developing insights about feedback systems.

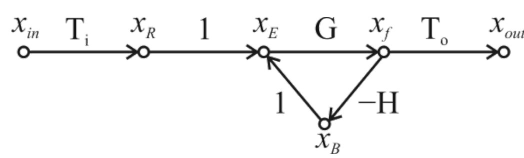

Figure 2.3. Flow-graph for single-loop feedback amplifier

The quantity x_{in} represents the input voltage or current into the system. The **error node**, x_E**,** is the node to which feedback is applied. The quantity x_R represents the **reference quantity** or the **open-loop error quantity**. This is the quantity at the error node if the feedback path is nulled. The quantity x_B is the **open-loop feedback quantity**. The nodes x_R and x_B add together to produce the error quantity, x_E. The quantity x_f is the current or voltage which is fed back to the error node. Finally, the node x_{out} is the voltage or current at the output of the system.

3. 2-port Feedback Network Theory

Understanding 2-port theory is useful for gaining some of the insights behind feedback analysis. Feedback loops generally fall into four topologies: series-series (voltage in, voltage out), series-shunt (voltage in, current out), shunt-series (current in, voltage out), and shunt-shunt (current in, current out). Commonly, *g* (shunt-series), *h* (series-shunt), *y* (shunt-shunt), and *z* (series-series) parameters are used to describe the gains and resistances in the two port models, although with our brief overview of 2-port networks, we will avoid discussing these parameters in any detail.

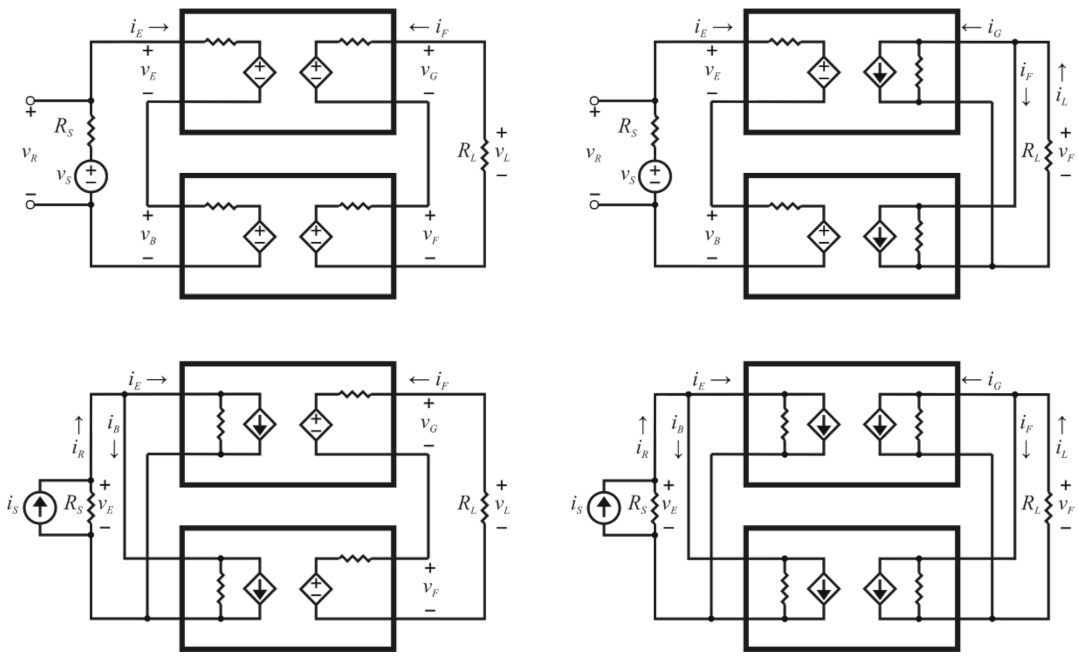

Figure 3.1. Four feedback network topologies: series-series (top left), series-shunt (top right), shunt-series (bottom left), and shunt-shunt (bottom right).

To apply 2-port feedback theory, we first decompose a feedback loop into two 2-port blocks, including an **open-loop amplifier** and a **feedback network**. Each 2-port block consists of two sources with a certain output resistance, one on the **error side** (left), and the other on the **feedback side** (right). In many cases, the dependent sources on the error side of the open-loop amplifier and on the feedback side are approximated to be zero. The source on the error side of each 2-port block is dependent on either i_F or v_F, either of which may be selected as the feedback nodes, and the source on the feedback side of each block is dependent on i_E or v_E, either of which may be selected as the error nodes. For series connections, it is more common select the current i_E or i_F as the node, and for a shunt connections, it is more common to select the voltage v_E or v_F as the node. When selecting as your node a voltage for the shunt connection or a current for the series connection, the dependent source inside the port across the voltage or through which the current flows has to be assumed zero.

Although 2-port feedback theory can provide us with insights into feedback theory, it has a tendency to overcomplicate the analysis, and it can often be impossible to decompose a feedback circuit into one of the four circuit topologies, especially in the case of the series-shunt or shunt-shunt topologies. For instance, consider the transconductance amplifier in Figure 3.2[1], consisting of a non-inverting op-amp configuration with discrete BJT output stage.

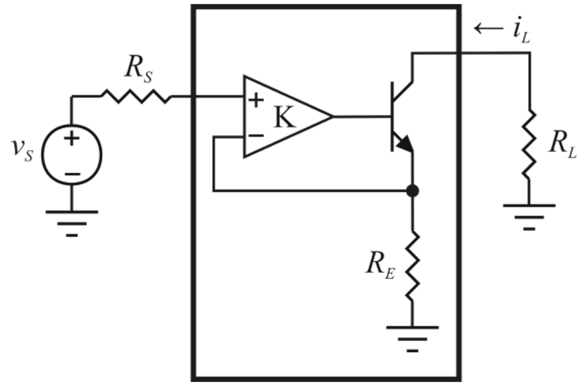

Figure 3.2. Transconductance amplifier

An erroneous attempt at fitting the transconductance amplifier in Figure 3.2 into the mold of the shunt-shunt feedback configuration will produce the following 2-port model.

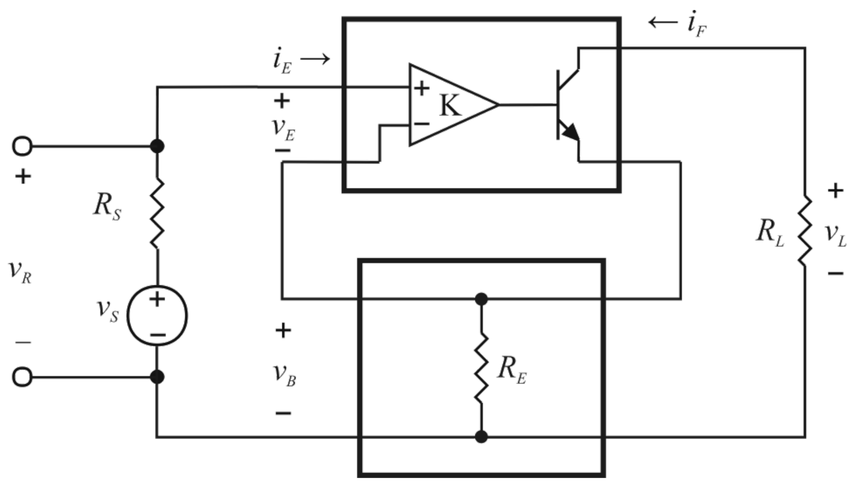

Figure 3.3. Erroneous application of 2-port series-series feedback theory

In Figure 3.3, the open-loop amplifier is chosen as the op-amp with the BJT output, and the feedback amplifier is chosen as the emitter resistor. It would appear at first that 2-port theory applies nicely in this situation. But since the currents into the collector and out of the emitter of the transistor are different, the 2-port model in Figure 3.3 breaks down. The 2-port model will work only if the current gain and r_o for the transistor are infinite; otherwise, the calculated output resistance of the open-loop amplifier will come out to be much higher than in the actual circuit. It turns out the circuit in Figure 3.2 will not fit into one of the 2-port molds unless the circuit is heavily modified.

[1] Taken from Jaegar & Blalock, page 1092

4. Identifying Feedback Nodes and Calculating Path Gains

The first step in applying feedback analysis to a feedback amplifier is to identify the feedback nodes and paths. This non-trivial task often requires a fair amount of circuit intuition. The goal for choosing the right nodes is to select an appropriate number of nodes for which the equations for the path gains between the nodes are fairly simple. In some circuits, the selection of nodes is obvious, while in other applications, there are multiple ways of selecting the nodes. It is generally easier to choose more nodes than necessary; then after drawing out a flow-graph diagram with the selected nodes, it is often easy to see which nodes are not really necessary and can be eliminated. Flow-graph reduction techniques can be then used to eliminate unnecessary nodes. Selecting too few or the incorrect feedback nodes will produce overly complicated node gain equations.

To begin, identify the nodes (voltages or currents) which seem to produce the most straight-forward paths. To do this, start with the input node, and take note of which of the other nodes are affected if the input node were to change in value. Then choose the node which seems to be most directly affected by changing the input quantity. Then repeat the process with the selected node, and continue until enough nodes are selected.

Next, identify the paths. To do this, we start with the input signal, x_{in}, and observe what effects a change on the value of this signal will have on other node quantities in the circuit. For a path to exist from node x_a to node x_b, changing node x_a will affect node x_b even when all of the other selected feedback nodes (x_c, x_d, x_e…) are set to zero. Keep in mind that no paths should lead to the input node since it is an ideal voltage or current source. During this process, it is helpful to construct a flow-graph diagram.

Finally, calculate the path gains. To calculate the gain of the path N from x_a to x_b, null all paths except for N which lead to the destination node x_b, and then determine the relation between x_a and x_b using circuit analysis techniques. If the relation between x_a and x_b is not simple, more nodes may need to be selected. To null a path, set the origin node quantity for the path to zero. Sometimes, multiple paths exist from x_a to x_b, in which case each path gain can be determined separately while the other paths are nulled, and then the individual path gains are added together to find the total path gain.

5. Input and Output Resistance Calculations for Feedback Circuits

As a result of feedback, the input and output resistance for amplifiers is affected. We can understand the resistance at a ground-referenced voltage node v_{out} by placing a test resistor R_{load} between the node and ground (see Figure 5.1). As a result of adding the resistor R_{load}, the voltage at the output node will drop from v_{out} by the amount Δv_{out}. The resistance at the node is therefore

$$R_{out} = R_{load}\left(\frac{\Delta v_{out}}{v_{out} - \Delta v_{out}}\right) \quad \textbf{(5.1)}$$

In a special case when Δv_{out} is half of v_{out}, we know that R_{in} and R_{load} are equal.

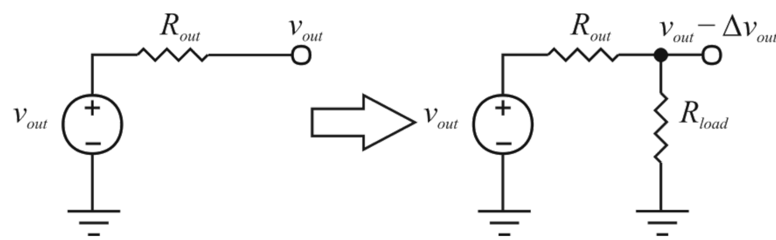

Figure 5.1. Test for resistance at a voltage node.

Similarly, we can understand the resistance along a path i_{out} by placing a test resistor across the resistance being measured (see Figure 5.2). As a result of adding the resistor R_{load}, the current through the output path will drop from i_{out} by the amount Δi_{out}. The resistance along the output path is therefore

$$R_{out} = R_{load}\left(\frac{\Delta i_{out}}{i_{out} - \Delta i_{out}}\right) \quad \textbf{(5.2)}$$

In a special case when Δi_{out} is half of i_{out}, we know that R_{in} and R_{load} are equal.

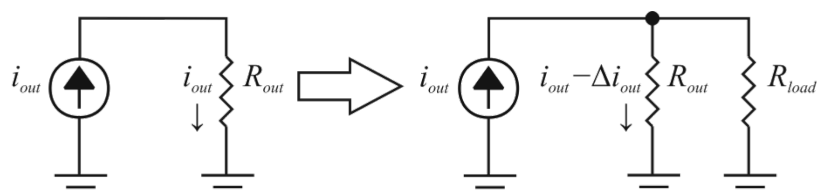

Figure 5.2. Test for resistance along a current path

When finding the input or output resistance of a circuit with feedback, we first need to know whether the node uses series or shunt feedback. Knowing this, we can then redraw the circuit as one of the following two circuits.

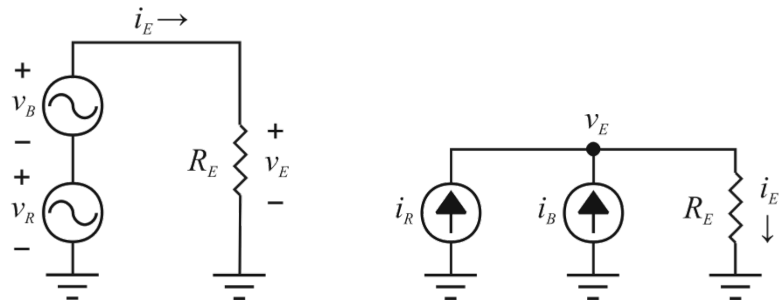

Figure 5.3. Test circuits for determining input resistance for series feedback (left) and shunt feedback (right)

Both of these diagrams are simplified circuit realizations of a single-loop feedback system (see Figure 2.3), one for a series connection (left) and the other for a shunt connection (right). Consider the series feedback circuit on the left. From the flow-graph diagram in Figure 2.3, we see that $v_E = v_R + v_B$, and $v_B = -G\,H\,v_E$. Also, we can see that the resistance seen by the v_R source is v_R / i_E. Therefore:

$$R_{in} = \frac{v_R}{i_E} = \frac{v_E - v_B}{i_E} = \frac{v_E\left(1+\mathrm{G\,H}\right)}{i_E} = R_E\left(1+\mathrm{G\,H}\right) \tag{5.3}$$

From this result, we see that if there is a series feedback connection at the selected error or feedback node, we can find the resistance along the path of the node if we zero the voltage v_B and calculate the open-loop resistance R_E. After finding the open-loop resistance, we can calculate the effective resistance seen along the path when feedback is applied by multiplying R_E by 1 + G H.

Similarly, for the circuit on the right, we find $i_E = i_R + i_B$, and from the flow-graph diagram, we see $i_B = -G\,H\,i_E$. We can also see that the resistance seen by the iR source is v_E / i_R. Therefore:

$$R_{in} = \frac{v_E}{i_R} = \frac{v_E}{i_E - i_B} = \frac{v_E}{i_E\left(1+\mathrm{G\,H}\right)} = \frac{R_E}{1+\mathrm{G\,H}} \tag{5.4}$$

From this result, we find that if there is a shunt connection at the selected error or feedback node, we can find the effective resistance looking into the node if we zero the current i_B and calculate the open-loop resistance R_E. After finding the open-loop resistance, we can calculate the effective resistance seen looking into the node when feedback is applied by dividing R_E by 1 + G H.

For all practical purposes, amplifiers are designed to be driven with a current or voltage source with finite output resistance, R_S, and are designed to drive a finite load with resistance R_L. When using equations (5.3) and (5.4) to find the input resistance into the input of a feedback node, it is generally useful to remove R_S or R_L before calculating the resistance looking into the node. When calculating the input resistance, we first remove the source resistor, and when calculating the output resistance, we first remove the load resistor. When the source or load circuitry is disconnected, this usually impacts the path gains in the circuit, so they would first need to be recalculated in the modified circuit before we could apply equations (5.3) and (5.4) to find the closed-loop resistance. Also, it is important to keep in mind that in most cases, the value of the source resistance is dependent on the load resistance, and vice versa.

If a voltage source with source resistance R_S drives a voltage error node v_E in a feedback loop, the input resistance can be found using ohm's law by dividing the source voltage by the source current. The source current is found by dividing the voltage drop across the source resistor by the source resistance R_S.

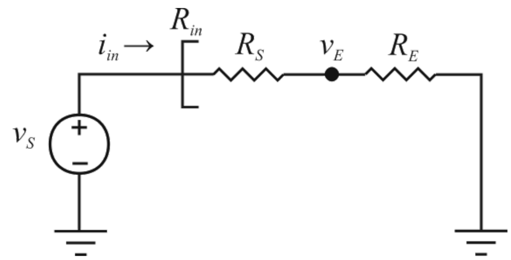

Figure 5.4. Test for input resistance using source and error voltages

$$R_{in} = \frac{v_S}{i_{in}} = \frac{v_S\,R_S}{v_S - v_E} = \frac{R_S}{1 - \frac{v_E}{v_S}} = \frac{R_S}{1 - \frac{\mathrm{T_i}}{1+\mathrm{GH}}} = R_S\left(\frac{1+\mathrm{GH}}{1+\mathrm{GH}-\mathrm{T_i}}\right) \quad \textbf{(5.5)}$$

To find the value of R_E only, we simply subtract the source resistance from the input resistance.

$$R_E = R_{in} - R_S = R_S\left(\frac{1+\mathrm{GH}}{1+\mathrm{GH}-\mathrm{T_i}} - 1\right) = R_S\left(\frac{\mathrm{T_i}}{1+\mathrm{GH}-\mathrm{T_i}}\right) \quad \textbf{(5.6)}$$

The same method can be used if a current source with resistance R_S drives a current error node i_E in a feedback loop. The input resistance can once again be found using ohm's law. The source voltage is found by multiplying the difference between the source current and error current by the source resistance.

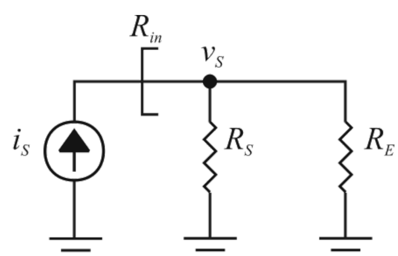

Figure 5.5. Test for input resistance using source and error currents

$$R_{in} = \frac{v_{in}}{i_S} = \frac{R_S\left(i_S - i_E\right)}{i_S} = R_S\left(1 - \frac{i_E}{i_S}\right) = R_S\left(1 - \frac{\mathrm{T_i}}{1+\mathrm{GH}}\right) = R_S\left(\frac{1+\mathrm{GH}-\mathrm{T_i}}{1+\mathrm{GH}}\right) \quad \textbf{(5.7)}$$

To find the value of R_E only, we first realize that R_{in} is R_S and R_E in parallel. Then, solving for R_E:

$$R_E = \frac{R_S\,R_{in}}{R_S - R_{in}} = \frac{R_S^2\left(\frac{1+\mathrm{GH}-\mathrm{T_i}}{1+\mathrm{GH}}\right)}{R_S\left(1 - \frac{1+\mathrm{GH}-\mathrm{T_i}}{1+\mathrm{GH}}\right)} = R_S\left(\frac{1+\mathrm{GH}-\mathrm{T_i}}{1+\mathrm{GH}+\mathrm{T_i}}\right) \quad \textbf{(5.8)}$$

In some cases such as the audio preamplifier circuit in section 10, there are nested feedback loops inside the main feedback loop. In these cases, finding the open-loop resistance into the circuit requires first finding the closed-loop resistance of intermediary feedback loops. The loop gain of the whole feedback can be found by reducing the flow-graph diagram to an equivalent single-path feedback loop, similar to Figure 2.1.

6. Resistive Divider Example

For our first example of a feedback system, we will analyze a voltage divider circuit consisting of three resistors, as shown in Figure 6.1. It may not be obvious at first, but the resistive voltage divider is the simplest examples of a feedback amplifier. We will apply the principles discussed in sections 4 and 5 to analyze the resistive network from a feedback perspective. Using feedback analysis on such a simple circuit is generally considered complete overkill, since any first-quarter electrical engineering student would be able to tell you immediately what the gain, input resistance, and output resistance of the above circuit on observation. Evaluating the circuit using feedback analysis will assist in gaining many of the critical insights required in feedback analysis.

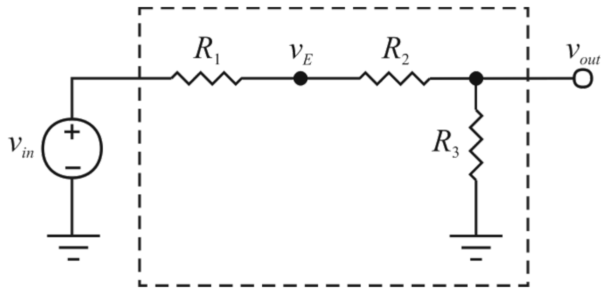

Figure 6.1. Resistive divider amplifier

To begin, we will choose v_{in}, v_E, and v_{out} as the feedback nodes. Next, we identify the paths between the nodes. For a path to exist from v_{in} to v_E, changing v_{in} will also cause v_E to change when v_{out} is grounded. And it can be easily seen that when v_{out} is grounded, v_E is still controlled by v_{in}, so a path exists from v_{in} to v_E. However, no path exists from v_{in} to v_{out}, since when the node v_E is grounded, v_{out} no longer changes with v_{in}. Using the same criteria, we can also see that paths exist from v_E to v_{out} and from v_{out} to v_E. We can now draw the following flow-graph of our system.

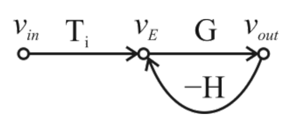

Figure 6.2. Flow-graph for resistive divider circuit

From this flow-graph, we see the following node path equations.

$$v_E = \mathrm{T_i}\, v_{in} - \mathrm{H}\, v_{out} \tag{6.1}$$

$$v_{out} = \mathrm{G}\, v_E \tag{6.2}$$

From the node path equations, we can solve for the node gain equations.

$$\frac{v_E}{v_{in}} = \frac{\mathrm{T_i}}{1+\mathrm{G\,H}} \tag{6.3}$$

$$\frac{v_{out}}{v_{in}} = \frac{\mathrm{T_i\,G}}{1+\mathrm{G\,H}} \tag{6.4}$$

The next step is to calculate the path gains. To calculate the path gain T_i, we first null all the other paths which lead to v_{in}, which includes the H path. To null the H path, we set the origin node for the H path, v_L, to zero. After doing this, we are left with a simple resistive divider.

$$T_i = \left.\frac{v_E}{v_{in}}\right|_{\text{H null}} = \frac{R_2}{R_1 + R_2} \quad \textbf{(6.5)}$$

To find the path gain G, we notice that v_{out} is the output of a simple 2-resistor resistive divider with input v_E.

$$G = \frac{v_{out}}{v_E} = \frac{R_3}{R_2 + R_3} \quad \textbf{(6.6)}$$

Finally, to find the path gain H, we have to first null the T_i path. We do this by setting v_{in} to zero. After doing this, we are yet again left with a simple resistive divider circuit.

$$H = -\left.\frac{v_E}{v_{out}}\right|_{T_i\text{ null}} = -\frac{R_1}{R_1 + R_2} \quad \textbf{(6.7)}$$

Finally, we plug the path gain equations into the node gain equations. After simplification, the node gain equations match the expected results after simply applying the voltage divider equations to the system.

$$\frac{v_E}{v_{in}} = \frac{\dfrac{R_2}{R_1 + R_2}}{1 + \left(\dfrac{R_3}{R_2 + R_3}\right)\left(-\dfrac{R_1}{R_1 + R_2}\right)} = \frac{R_2 + R_3}{R_1 + R_2 + R_3} \quad \textbf{(6.8)}$$

$$\frac{v_{out}}{v_{in}} = \frac{\left(\dfrac{R_2}{R_1 + R_2}\right)\left(\dfrac{R_3}{R_2 + R_3}\right)}{1 + \left(\dfrac{R_3}{R_2 + R_3}\right)\left(-\dfrac{R_1}{R_1 + R_2}\right)} = \frac{R_3}{R_1 + R_2 + R_3} \quad \textbf{(6.9)}$$

To complete the 2-port model of the system, we need to know the input and output resistance for the circuit. Using the loop gain, G H, we can determine the resistance looking into the v_E and v_{out} nodes. To find the resistance looking into the node v_E, we first null all paths which have as their destination node v_E, which includes the T_i and H paths. To do this, we set the nodes v_{in} and v_{out} to zero. Next, we find the open-loop resistance looking into the node, which is $R_1 \parallel R_2$. Finally, we divide the open-loop resistance by one plus the loop gain, or 1 + G H.

$$R_E = \frac{R_1 \parallel R_2}{1 + \mathrm{G\,H}} = \frac{\dfrac{R_1 R_2}{R_1 + R_2}}{1 + \left(\dfrac{R_3}{R_2 + R_3}\right)\left(-\dfrac{R_1}{R_2 + R_1}\right)} = \frac{R_1\left(R_2 + R_3\right)}{R_1 + R_2 + R_3} = R_1 \parallel \left(R_2 + R_3\right) \quad \textbf{(6.10)}$$

It is more useful for us to find the resistance looking into the input port of the amplifier. To do this, we can do one of two things. One option is to find the limit of the error node input resistance in equation

(6.10) as R_1 goes to infinity, then add R_1 to the results. The other option is to use equation (5.5), using $R_S = R_1$.

$$R_{in} = R_1\left(\frac{1+\mathrm{G\,H}}{1+\mathrm{G\,H}-\mathrm{T_i}}\right) = R_1 \cdot \frac{1+\left(\frac{R_3}{R_2+R_3}\right)\left(-\frac{R_1}{R_1+R_2}\right)}{1+\left(\frac{R_3}{R_2+R_3}\right)\left(-\frac{R_1}{R_1+R_2}\right)-\frac{R_2}{R_1+R_2}} \quad \textbf{(6.11)}$$

$$\rightarrow R_{in} = R_1 + R_2 + R_3 \quad \textbf{(6.12)}$$

To find the output resistance looking into the node v_{out}, we first null the G path by setting v_{in} to zero. We end up with an open-loop output resistance of $R_2 \parallel R_3$. Using equation (5.4), we then divide this value by 1 + G H.

$$R_{out} = \frac{R_2 \parallel R_3}{1+\mathrm{G\,H}} = \frac{\frac{R_2\,R_3}{R_2+R_3}}{1+\left(\frac{R_3}{R_2+R_3}\right)\left(-\frac{R_1}{R_2+R_1}\right)} = \frac{R_3\left(R_1+R_2\right)}{R_1+R_2+R_3} = \left(R_1+R_2\right) \parallel R_3 \quad \textbf{(6.13)}$$

In this example, we turned a very simple problem into a complicated one by using feedback analysis. For more complicated feedback systems, the use of feedback analysis can go a long way to providing insights into feedback systems.

7. Inverting Amplifier Example

In this example, we will analyze an inverting amplifier with an ideal op-amp with finite gain K.

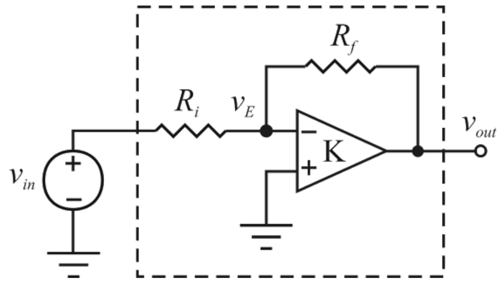

Figure 7.1. Inverting op-amp circuit

To begin, we will select for our nodes v_{in}, v_E, and v_{out}. We can find node path equations for v_E and v_{out} by superposition:

$$v_E = v_{in} \cdot \frac{R_f}{R_f + R_i} + v_{out} \cdot \frac{R_i}{R_f + R_i} \tag{7.1}$$

$$v_{out} = -\mathrm{K}\, v_E \tag{7.2}$$

A flow-graph of the finite-gain inverting amplifier can be constructed from these two equations, which is depicted in Figure 7.2.

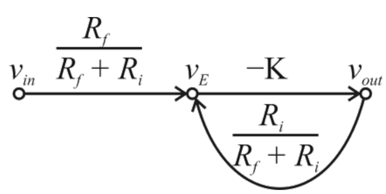

Figure 7.2. Flow-graph for finite-gain inverting amplifier

Our system therefore has the same form as the typical single-path amplifier depicted in Figure 2.1 if we choose for our nodes $x_{in} = v_{in}$, $x_{out} = x_f = v_{out}$, and $x_E = v_E$. The path gains are therefore $\mathrm{T_i} = R_f / (R_f + R_i)$, G = -K, H = $-R_i / (R_f + R_i)$, and $\mathrm{T_o} = 1$. Solving the node path equations, we produce the node gain equations:

$$\frac{v_E}{v_{in}} = \frac{R_f}{R_f + R_i(\mathrm{K}+1)} \tag{7.3}$$

$$\frac{v_{out}}{v_{in}} = -\frac{\mathrm{K}\, R_f}{R_f + R_i(\mathrm{K}+1)} \tag{7.4}$$

From equation 5b, we see that as K approaches infinity, the gain of the inverting op-amp approaches – R_f / R_i, which is the gain of an ideal inverting op-amp circuit.

The input resistance of the inverting op-amp is affected by feedback. To find the input resistance of the circuit, we can use either of two approaches. The first approach is to find the limit as R_i becomes infinitely large of the input resistance looking into the node v_E using equation (5.4). We first null the feedback path by grounding v_{out}. We then find the open-loop input resistance looking into the node v_E, which is $R_i \parallel R_f$. Then, applying equation (5.4), we divide the results by 1 + G H to find the closed-loop resistance R_E.

$$R_E = \frac{R_i \parallel R_f}{1 + \mathrm{G\,H}} = \frac{R_i \parallel R_f}{1 + (-\mathrm{K})\left(-\frac{R_i}{R_f + R_i}\right)} = \frac{R_f\, R_i}{R_f + R_i(\mathrm{K}+1)} \tag{7.5}$$

The input resistance can be found by finding the limit of equation (7.5) as R_i goes to infinity, then adding R_i to the results.

$$R_{in} = R_i + \lim_{R_i \to \infty}(R_E) = R_i + \frac{R_f}{\mathrm{K}+1} \tag{7.6}$$

The other approach to finding the input resistance is to use equation (5.5), using $R_S = R_i$.

$$R_{in} = R_i\left(\frac{1 + \mathrm{G\,H}}{1 + \mathrm{G\,H} - \mathrm{T_i}}\right) = R_i \cdot \frac{1 + (-\mathrm{K})\left(-\frac{R_i}{R_f + R_i}\right)}{1 + (-\mathrm{K})\left(-\frac{R_i}{R_f + R_i}\right) - \frac{R_f}{R_f + R_i}} = R_i + \frac{R_f}{\mathrm{K}+1} \tag{7.7}$$

For this example, we are assuming the op-amp has ideal output resistance; therefore, the output resistance for the inverting amplifier is also zero.

8. Discrete Component BJT Example

In this example, we will analyze the following discrete-component BJT amplifier. For this analysis, we will assume Q_1 and Q_2 have the same current gain, β. The amplifier being analyzed is within the dotted rectangle, and the resistors R_S and R_L represent the output resistance of the previous stage and the input resistance of the following stage. We will use feedback analysis to determine the paths, and overall transresistance of the amplifier. The first step is to determine the DC operating points; although, for this example, we will skip the static analysis.

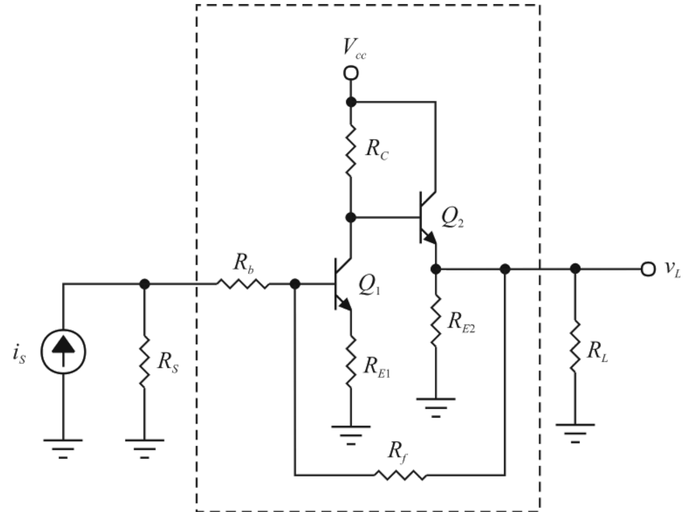

Figure 8.1. Discrete-component BJT feedback amplifier

For the static analysis, we will use the BJT T-model, and assume that r_o is negligible. The resulting quasi-static model is shown in Figure 8.2.

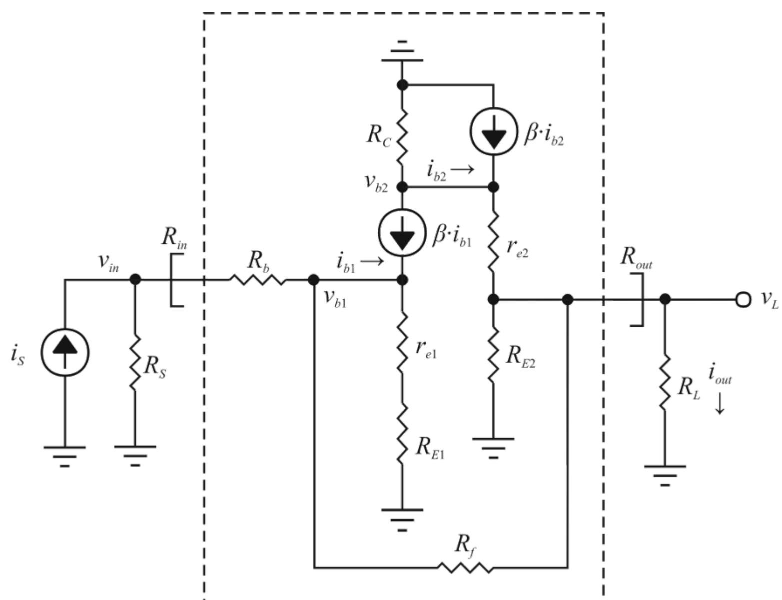

Figure 8.2. Quasi-static model of discrete-component BJT feedback amplifier

To begin, we will identify the feedback nodes. The first two feedback nodes we will select are the input and output nodes, i_S and v_L. Although, it isn't at all clear which other circuit quantities to choose. Some of the possible candidates for feedback nodes are v_{b1}, i_{b1}, v_{b2}, i_{b2}, and v_{in}. It isn't clear how many of these potential feedback nodes we will need to perform the feedback analysis. For this example, we will perform an experiment, and choose all of the above listed nodes as feedback nodes. After identifying the paths, we come up with the following flow-graph diagram for the system.

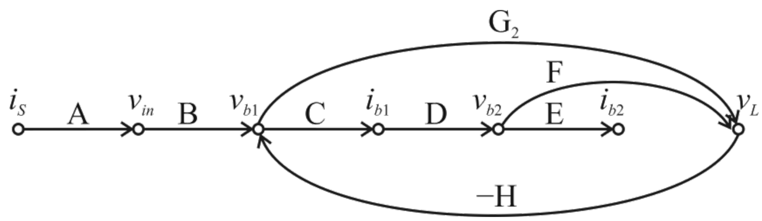

Figure 8.3. Flow-graph of discrete-component BJT feedback amplifier

We see that we can drastically simplify the feedback system. First, we can clearly discard i_{b2} as a feedback node. We can combine paths which lie in series, such as A B and C D F, by discarding the nodes separating them (v_{in}, i_{b1}, v_{b2}, and i_{b2}) and multiplying the path gains. Doing this, we can reduce the feedback system reduces to the typical single-path feedback amplifier, similar to Figure 2.1, with T_i = A B, G_1 = C D F, G = G_1+ G_2, and T_o = 1.

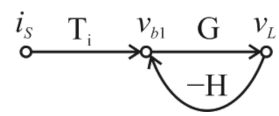

Figure 8.4. Reduced flow-graph of BJT feedback amplifier

Next, we calculate the feedback gains. To find the transresistance T_i, we must null the H path by grounding v_L. At this point, it helps to transform the current source on the input into the Thévenin equivalent voltage source with voltage $i_{in} R_{in}$ and input resistance R_{in}. We can then find T_i using the voltage divider equation.

$$\mathrm{T_i} = \left.\frac{v_{b1}}{i_S}\right|_{\mathrm{H\ null}} = R_S \cdot \frac{R_f \,||\left[(\beta+1)(r_{e1}+R_{E1})\right]}{R_S + R_b + R_f \,||\left[(\beta+1)(r_{e1}+R_{E1})\right]} \qquad \textbf{(8.1)}$$

Next, we find G, which is v_L / v_{b1}. We note that G takes on two separate paths: G_1, which goes through the transistors; and G_2, which goes through R_f. To find the gain G_1, we first must null the contribution through the G_2 path by disconnecting the left led of R_f from the base of Q_1 and grounding it. The gain of the common-emitter transistor is:

$$\mathrm{G}_{1a} = \left.\frac{v_{b2}}{v_{b1}}\right|_{\mathrm{G_2\ null}} = -\alpha \cdot \frac{R_C \,||\left[(\beta+1)(r_{e2}+R_{E2} \,||\, R_L \,||\, R_f)\right]}{r_{e1}+R_{E1}} \qquad \textbf{(8.2)}$$

The gain of the emitter-follower output stage is:

$$G_{1b} = \left.\frac{v_L}{v_{b2}}\right|_{\mathrm{G}_2\ \mathrm{null}} = \frac{R_{E2} \,\|\, R_L \,\|\, R_f}{r_{e2} + R_{E2} \,\|\, R_L \,\|\, R_f} \quad \textbf{(8.3)}$$

Therefore, the gain G_1 is:

$$\mathrm{G}_1 = \left.\frac{v_L}{v_{b1}}\right|_{\mathrm{G}_2\ \mathrm{null}} = G_{1a}\, G_{1b} \quad \textbf{(8.4)}$$

We will now find G_2. We do this by nulling the contribution through G_1 by disconnecting and grounding the base of Q_1. Using a voltage divider equation, we can then find G_2.

$$\mathrm{G}_2 = \left.\frac{v_L}{v_{b1}}\right|_{\mathrm{G}_1\ \mathrm{null}} = \frac{R_{E2} \,\|\, R_L \,\|\, \left[r_{e2} + R_C/(\beta+1)\right]}{R_f + R_{E2} \,\|\, R_L \,\|\, \left[r_{e2} + R_C/(\beta+1)\right]} \quad \textbf{(8.5)}$$

We can now find the path gain G by adding G_1 and G_2. Simplifications can be made by assuming the gain of the emitter-follower is 1 or ignoring the G_2 path, which more than likely is insignificant compared to G_1.

To find the H path gain, we must first null the $\mathrm{T_i}$ path by setting i_{in} to zero. We can then use a voltage divider equation to calculate the path gain.

$$\mathrm{H} = -\left.\frac{v_{b1}}{v_L}\right|_{\mathrm{T_i\ null}} = -\frac{(R_b + R_S) \,\|\, \left[(\beta+1)(r_{e1} + R_{E1})\right]}{R_f + (R_b + R_S) \,\|\, \left[(\beta+1)(r_{e1} + R_{E1})\right]} \quad \textbf{(8.6)}$$

Because the path gain equations for our system turned out quite complicated, we won't even bother to find the total transresistance of our amplifier since the equation probably won't fit on this page.

To find the input resistance, we can use equation (5.6), although first, we must convert the input source into the Thévenin equivalent voltage source. Because of this transformation, $\mathrm{T_i}$ gets divided by R_S.

$$R_{in} = R_b + (R_S + R_b)\left(\frac{\frac{\mathrm{T_i}}{R_S}}{1 + \mathrm{G\,H} - \frac{\mathrm{T_i}}{R_S}}\right) \quad \textbf{(8.7)}$$

The other way of finding the input resistance for single-path feedback amplifiers is to use equation (5.4) after letting R_b go to infinity. Setting R_b to infinity causes the path gain H to become:

$$\mathrm{H}\big|_{R_b=\infty} = -\frac{(\beta+1)(r_{e1} + R_{E1})}{R_f + (\beta+1)(r_{e1} + R_{E1})} \quad \textbf{(8.8)}$$

To find the open-loop input resistance, disconnect and ground the right led of R_f.

To find the output resistance, we can once again use equation (5.4), although we must first null the output current by letting R_L go to infinity. This changes the path gains G_{1a}, G_{1b}, and G_2. After removing the load, these path gains become:

$$\mathrm{G}_{1a}\big|_{R_L\to\infty} = -\alpha \cdot \frac{R_C \parallel \left[(\beta+1)\left(r_{e2} + R_{E2} \parallel R_f\right)\right]}{r_{e1} + R_{E1}} \quad \textbf{(8.9)}$$

$$\mathrm{G}_{1b}\big|_{R_L\to\infty} = \frac{R_{E2} \parallel R_f}{r_{e2} + R_{E2} \parallel R_f} \quad \textbf{(8.10)}$$

$$\mathrm{G}_2\big|_{R_L\to\infty} = \frac{R_{E2} \parallel \left[r_{e2} + R_C/(\beta+1)\right]}{R_f + R_{E2} \parallel \left[r_{e2} + R_C/(\beta+1)\right]} \quad \textbf{(8.11)}$$

We then find the closed-loop output resistance by finding the open-loop resistance after R_L has been removed, and applying equation (5.4).

$$R_{out} = \frac{R_f \parallel R_{E2} \parallel \left[r_{e2} + R_C/(\beta+1)\right]}{1 + \mathrm{G\,H}\big|_{R_L\to\infty}} \quad \textbf{(8.12)}$$

9. Shunt-feedback Amplifier Example

A commonly-used feedback amplifier is the shunt-feedback amplifier. This circuit consists of a common-emitter amplifier with a resistor shunting the base and collector pins.

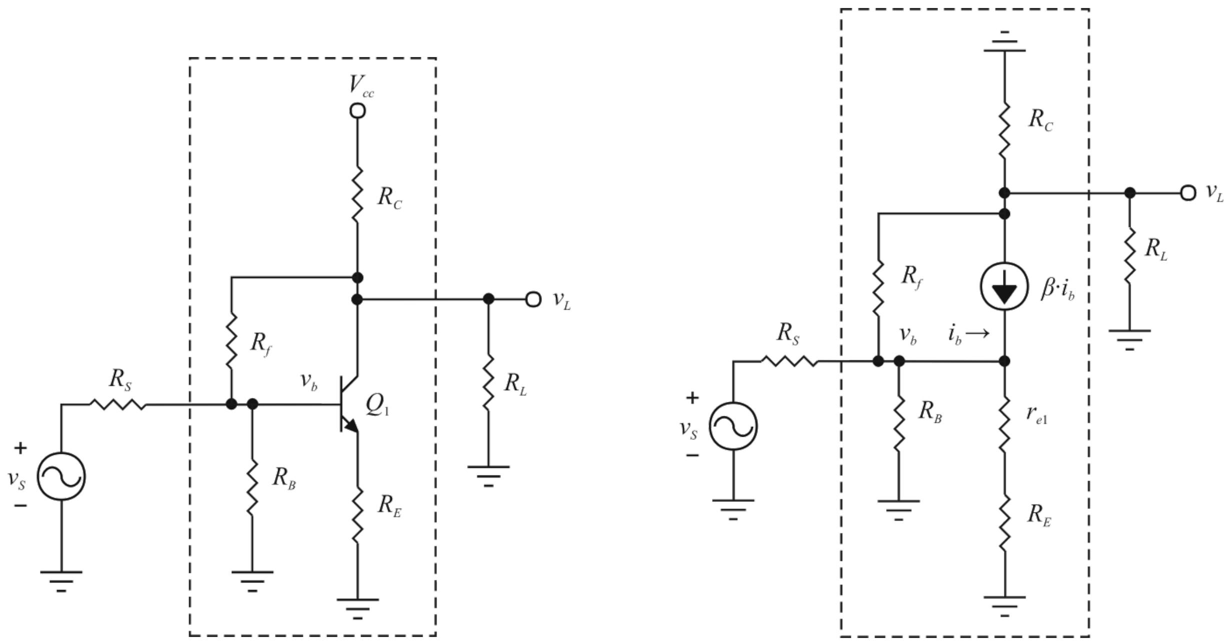

Figure 9.1. Shunt-feedback Amplifier, static (left) and quasi-static (right) models

We begin by choosing v_S, v_b, and v_L as nodes. The flow-graph looks as follows:

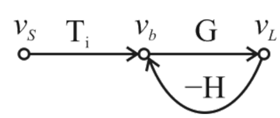

Figure 9.2. Flow-graph for shunt-feedback amplifier

We will let r_B be the resistance looking into the base of the transistor. Next, we find the path gains. To find T_i, we first null the H path by setting v_L to zero.

$$T_i = \left.\frac{v_b}{v_S}\right|_{\text{H null}} = \frac{R_f \parallel R_B \parallel r_B}{R_S + R_f \parallel R_B \parallel r_B} \tag{9.1}$$

The G path consists of two different paths, G_1 and G_2. To find G_1, we null G_2 by disconnecting and grounding the base of the transistor. To find G_2, we null the forward path through R_f by disconnecting and grounding the bottom led of R_f. Finally, we find the total path gain by adding G_1 and G_2 together.

$$G_1 = \left.\frac{v_L}{v_b}\right|_{\text{G2 null}} = \frac{R_C \parallel R_L}{R_f + R_C \parallel R_L} \tag{9.2}$$

$$G_2 = \left.\frac{v_L}{v_b}\right|_{G_1\ \text{null}} = -\beta \cdot \frac{R_C \,||\, R_L \,||\, R_f}{r_B} \quad \textbf{(9.3)}$$

$$G = \frac{v_L}{v_b} = G_1 + G_2 = \frac{R_C \,||\, R_L}{R_f + R_C \,||\, R_L} - \beta \cdot \frac{R_C \,||\, R_L \,||\, R_f}{r_B} \quad \textbf{(9.4)}$$

For the H path, we first null the T_i path by grounding v_S.

$$H = -\left.\frac{v_b}{v_L}\right|_{T_i\ \text{null}} = -\frac{R_S \,||\, R_B \,||\, r_B}{R_f + R_S \,||\, R_B \,||\, r_B} \quad \textbf{(9.5)}$$

Finally, we find the input and output resistance. To find the input resistance, we can apply equation (5.6). Using this method, the input resistance reduces to:

$$R_{in} = R_S + R_B \,||\, r_B \left\|\left(\frac{R_f + R_C \,||\, R_L}{\beta \cdot \frac{R_C \,||\, R_L}{r_B} + 1} \right)\right. \quad \textbf{(9.6)}$$

For the output resistance, we first remove R_L, which affects the G path gain. We then apply equation (5.4). The results reduce to:

$$R_{out} = R_C \left\|\left(\frac{R_f + R_S \,||\, R_B \,||\, r_B}{\beta \cdot \frac{R_S \,||\, R_B \,||\, r_B}{r_B} + 1} \right)\right. \quad \textbf{(9.7)}$$

In equations (9.6) and (9.7), we see an interesting insight on the effects of feedback to the resistance looking through the feedback resistor, R_f. In both cases, we end up with equations which divide the open-loop resistance through the feedback resistor by one plus beta times the ratio of resistance past the feedback resistor to the resistance looking into the base of the transistor.

10. Multiple Feedback Audio Pre-amplifier Example

In this example, we will use feedback analysis techniques to evaluate a multiple-feedback circuit[2]. The circuit, seen in Figure 10.1, is an audio pre-amplifier circuit with non-inverting feedback through R_f. The circuit consists of two shunt-feedback amplifiers in series. The output of the second shunt-feedback amplifier stage feeds back to the emitter of the first stage through a feedback resistor. The output of the second stage is buffered by an emitter-follower.

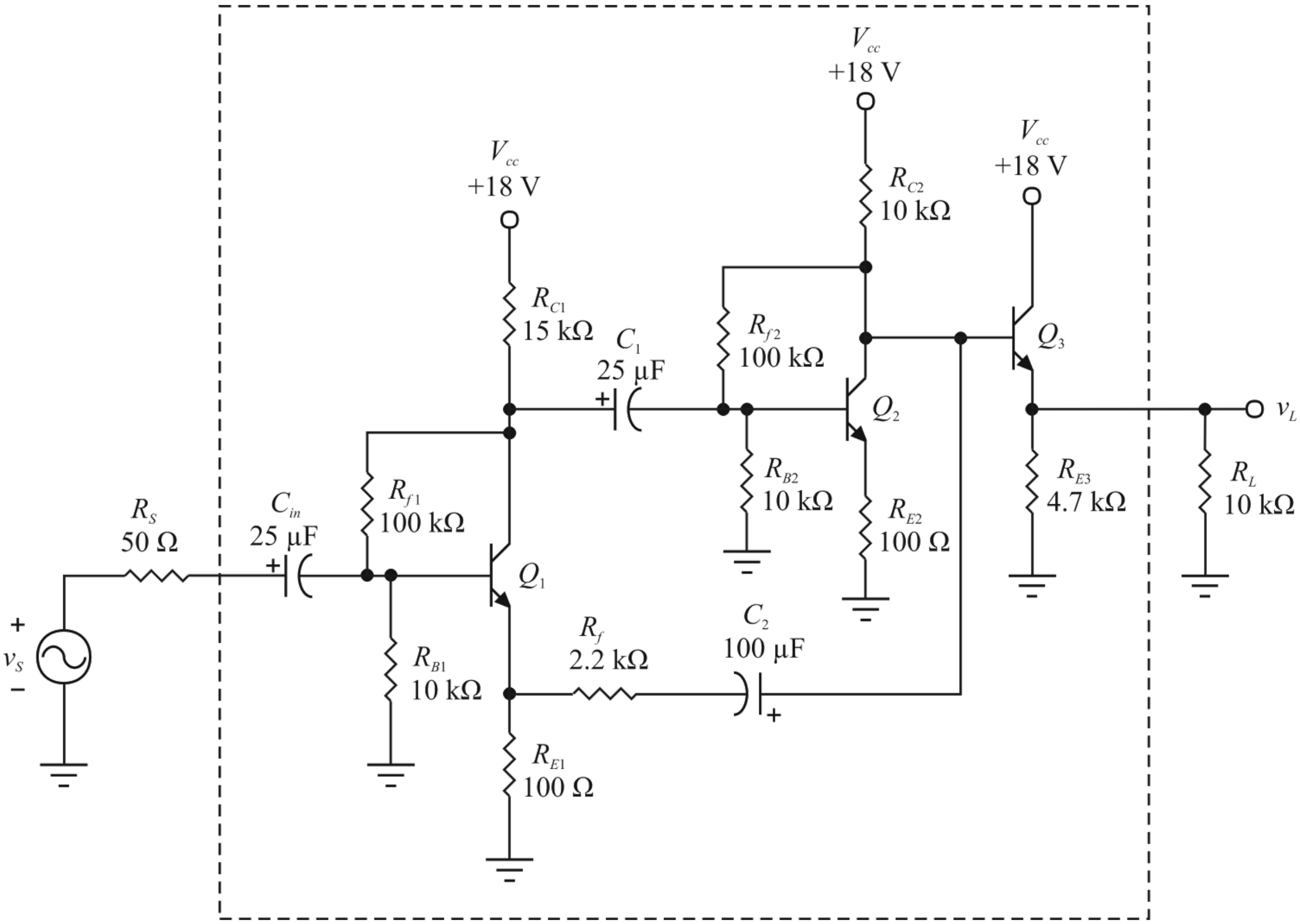

Figure 10.1. Audio Preamplifier with Non-inverting Feedback

For this analysis, we will use find the quasi-static gain for the above circuit, as well as the input and output resistance. For the static analysis, we will assume the circuit is operating at room temperature (V_T = 25.8 mV), the resistors are all the same type with saturation current, I_s, of 15 fA, and current gain, β, of 99 A/A. We will also assume a source resistance of R_S = 50 Ω and a load resistance of R_L = 10 kΩ. Using static analysis, we find the operating point parameters for the transistors are r_{e1} = 43.90 Ω,

[2] Taken from Feucht, Page 206

r_{e2} = 30.95 Ω, and r_{e3} = 14.95 Ω. A quasi-static model of the circuit using the BJT T-model is shown in Figure 10.2.

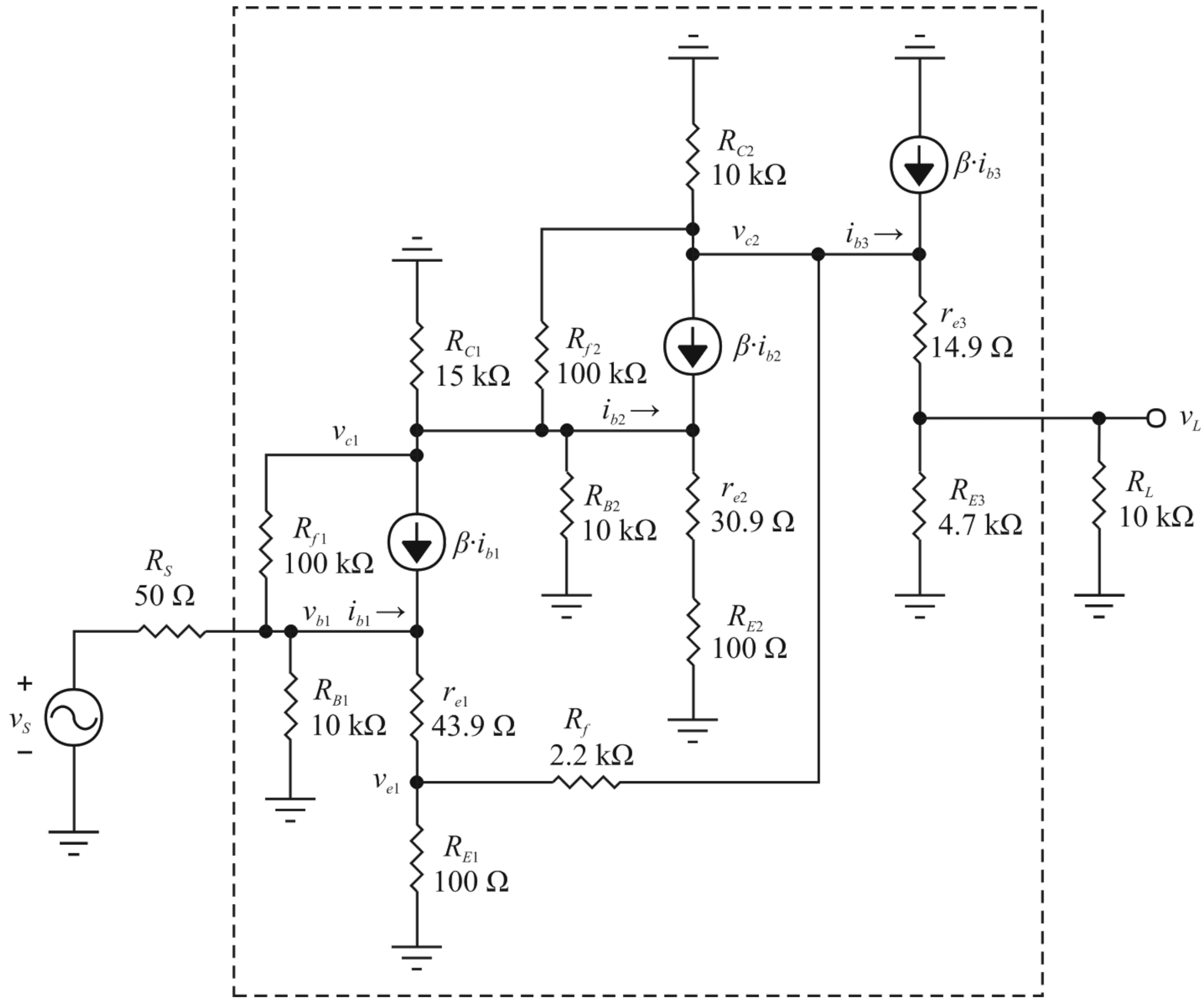

Figure 10.2. Quasi-static model of multiple-path feedback amplifier

We begin by selecting the feedback nodes in the circuit. For this circuit, we will choose v_S, v_{b1}, i_{b1}, v_{c1}, v_{c2}, and v_L as the nodes. The next step is to identify what the paths in the circuit are. We can see that changing v_{in} has a direct affect on the current i_{b1} and the voltage v_{c1}. We also see that changing i_{b1} has a direct affect on v_{c1}. On first glance, one might be led to believe that a path exists from i_{b1} to v_{c2}. For such a direct path to exist, a change in i_{b1} would affect v_{e2} when the nodes v_{in} and v_{c1} are zeroed; however, zeroing these nodes also zeroes i_{b1}, so no such path exists. A path does exist from v_{in} to v_{c2}. After finding all the paths in the circuit, we can create a flow-graph of the system, which is shown in Figure 10.3.

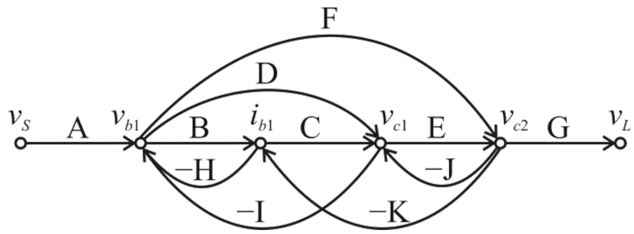

Figure 10.3. Flow-graph diagram of audio preamplifier

We will make the following definitions to make the feedback analysis simpler.

$$r_{b1} = (\beta+1)\left[r_{e1} + R_{E1} \parallel \left(R_f + r_{b3} \parallel R_{C2} \parallel R_{f2}\right)\right] \tag{10.1}$$

$$r_{b2} = (\beta+1)(r_{e2} + R_{E2}) \tag{10.2}$$

$$r_{b3} = (\beta+1)(r_{e3} + R_{E3} \parallel R_L) \tag{10.3}$$

To find the path gain A, we null the H path by setting v_{c1} to zero.

$$A = \left.\frac{v_{b1}}{v_S}\right|_{\text{H, I null}} = \frac{r_{b1} \parallel R_{B1} \parallel R_{f1}}{R_S + r_{b1} \parallel R_{B1} \parallel R_{f1}} = 994.5 \text{ mV/V} \tag{10.4}$$

To find the path gain B, we null the K path by setting v_{c2} to zero, and then apply Ohm's law.

$$B = \left.\frac{i_{b1}}{v_{b1}}\right|_{\text{K null}} = \frac{1}{(\beta+1)\left(r_{e1} + R_{E1} \parallel R_f\right)} = 71.66 \ \mu\text{A/V} \tag{10.5}$$

To find the path gain C, we null the D and J paths by zeroing v_{b1} and v_{c2}, and then apply Ohm's law.

$$C = \left.\frac{v_{c1}}{i_{b1}}\right|_{\text{D, J null}} = -\beta\left(R_{f1} \parallel R_{C1} \parallel R_{f2} \parallel r_{b2} \parallel R_{B2}\right) = -376.6 \text{ V/mA} \tag{10.6}$$

To find the path gain D, we null the C and J paths by zeroing i_{b1} and v_{c2}, and then apply the voltage divider equation.

$$D = \left.\frac{v_{c1}}{v_{b1}}\right|_{\text{C, J null}} = \frac{R_{C1} \parallel R_{f2} \parallel r_{b2} \parallel R_{B2}}{R_{f1} + R_{C1} \parallel R_{f2} \parallel r_{b2} \parallel R_{B2}} = 38.04 \text{ mV/V} \tag{10.7}$$

To find E, we have to consider two separate paths from v_{c1} to v_{c2}, which we will call E_1 and E_2. The path E_1 goes through Q_2, and the path E_2 goes through R_{f2}. To find the path gain for one of these paths, the contribution of v_{c2} from the other path must first be nulled. To null E_1, the bottom led of resistor R_{f2} is disconnected from node v_{c1} and grounded. To null E_2, the base of the transistor Q_2 is disconnected from node v_{c1} and grounded. The path gain E is the sum of the gains for the two individual paths, $E_1 + E_2$. To find either path, the F path must be nulled by setting v_{b1} to zero. The path gain E_1 is solved for using the transistor gain equation for a common emitter amplifier. The path gain E_2 is found using the voltage divider equation.

$$E_1 = \left.\frac{v_{c2}}{v_{c1}}\right|_{E_2,\ F\ null} = -\alpha \cdot \frac{R_{L2} \parallel R_{f2} \parallel \left(R_f + R_{E1} \parallel r_{e1}\right) \parallel r_{b3}}{r_{e2} + R_{E2}} = -13.44\ \text{V/V} \quad \textbf{(10.8)}$$

$$E_2 = \left.\frac{v_{c2}}{v_{c1}}\right|_{E_1,\ F\ null} = \frac{R_{C2} \parallel \left(R_f + R_{E1} \parallel r_{e1}\right) \parallel r_{b3}}{R_{f2} + R_{C2} \parallel \left(R_f + R_{E1} \parallel r_{e1}\right) \parallel r_{b3}} = 17.81\ \text{mV/V} \quad \textbf{(10.9)}$$

$$\rightarrow E = E_1 + E_2 = -13.42\ \text{V/V} \quad \textbf{(10.10)}$$

To solve for the path gain F, we null the E path by grounding v_{c1}. In this case, it becomes useful to Thévenize a portion of the circuit to make analysis more intuitive. This transformation is shown in Figure 10.4. The path gain is then found by applying the voltage divider equation.

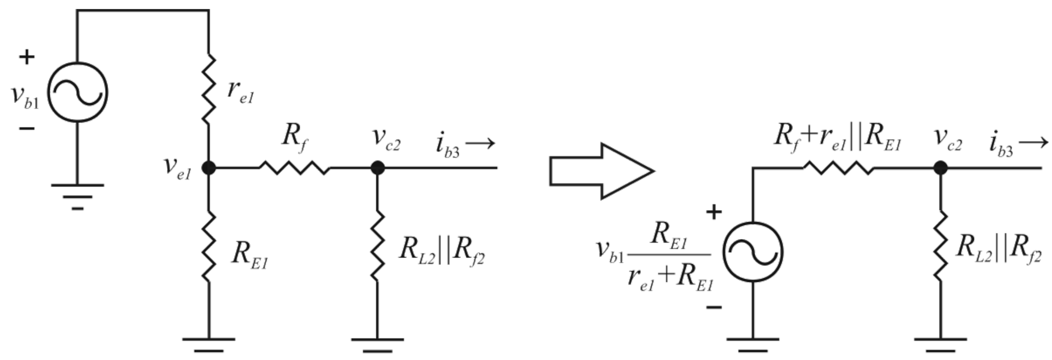

Figure 10.4. Thévenin transformation used to find path gain F

$$F = \left.\frac{v_{c2}}{v_{in}}\right|_{E\ null} = \frac{R_{E1}}{r_{e1} + R_{E1}} \cdot \frac{R_{C2} \parallel R_{f2} \parallel r_{b3}}{R_f + r_{e1} \parallel R_{E1} + R_{C2} \parallel R_{f2} \parallel r_{b3}} = 554.9\ \text{mV/V} \quad \textbf{(10.11)}$$

To find the path gain G, we apply the voltage divider equation.

$$G = \frac{v_L}{v_{c2}} = \frac{R_{E3} \parallel R_L}{r_{e3} + R_{E3} \parallel R_L} = 996.8\ \text{mV/V} \quad \textbf{(10.12)}$$

To find the path gain H, we null the A and I paths by zeroing v_S and v_{e1}, and then apply Ohm's law.

$$H = -\left.\frac{v_{c1}}{v_{c2}}\right|_{A,\ I\ null} = R_S \parallel R_{B1} \parallel R_{f1} = 49.73\ \text{V/V} \quad \textbf{(10.13)}$$

To find the path gain I, we null the A and H paths by zeroing v_S and i_{b1}, and then apply the voltage divider equation.

$$I = -\left.\frac{v_{b1}}{v_{c1}}\right|_{A,\ H\ null} = -\frac{R_S \parallel R_{B1}}{R_{f1} + R_S \parallel R_{B1}} = -497.3\ \mu\text{A/V} \quad \textbf{(10.14)}$$

To find the path gain J, we null the C and D paths by zeroing v_{b1} and i_{b1}, and then apply the voltage divider equation.

$$\mathrm{J} = -\left.\frac{v_{c1}}{v_{c2}}\right|_{\text{C, D null}} = -\frac{R_{f1} \parallel R_{C1} \parallel R_{B2} \parallel r_{b2}}{R_{f2} + R_{f1} \parallel R_{C1} \parallel R_{B2} \parallel r_{b2}} = -38.04 \text{ mV/V} \tag{10.15}$$

To find the path gain K, we null the B path by zeroing v_{b1}. We then Thévenize v_{c2} using the same technique we used for finding the F path gain, then apply Ohm's law.

$$\mathrm{K} = -\left.\frac{i_{b1}}{v_{c2}}\right|_{\text{B null}} = \frac{R_{E1}}{R_f + R_{E1}} \cdot \frac{1}{(\beta+1)\left(r_{e1} + R_f \parallel R_{E1}\right)} = 3.115 \ \mu\text{A/V} \tag{10.16}$$

For such a complicated feedback system, it is helpful to reduce the flow diagram to a typical single-path feedback amplifier. To begin, let's collapse the i_{b1} node.

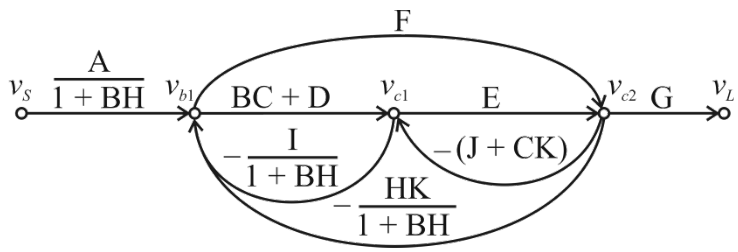

Figure 10.5. Feedback system after collapsing i_{b1} node

In the above figure, we see that our feedback system consists of two distinguishable intermediary feedback loops. The first loop is the first shunt-feedback stage, consisting of nodes v_{b1} and v_{c1}. The second loop is the second shunt-feedback stage, consisting of nodes v_{c1} and v_{c2}. When finding the open-loop output resistance at the nodes v_{b1} and v_{c2} for the entire system, we will first need to calculate the closed-loop resistance into the intermediate feedback loops.

We can further reduce the system by removing the node v_{c1}. Doing this, we result in a flow-graph diagram of a single-path feedback amplifier similar to the flow-graph in Figure 2.1.

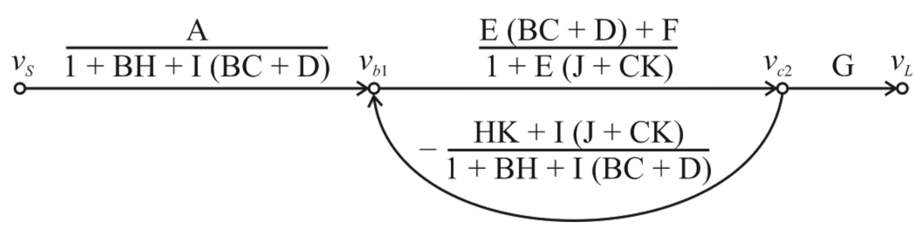

Figure 10.6. Feedback system converted into equivalent single-feedback path model

$$\mathrm{T_i} = \left.\frac{v_{b1}}{v_S}\right|_{\mathrm{T_h\ null}} = \frac{\mathrm{A}}{1 + \mathrm{BH} + \mathrm{I}(\mathrm{BC} + \mathrm{D})} = 99.34 \text{ mV/V} \tag{10.17}$$

$$\mathrm{T_g} = \frac{v_{c2}}{v_{b1}} = \frac{\mathrm{E}(\mathrm{BC} + \mathrm{D}) + \mathrm{F}}{1 + \mathrm{E}(\mathrm{J} + \mathrm{CK})} = 20.99 \text{ V/V} \tag{10.18}$$

$$\mathrm{T_h} = -\left.\frac{v_{b1}}{v_{c2}}\right|_{\mathrm{T_i\ null}} = -\frac{\mathrm{HK} + \mathrm{I}(\mathrm{J} + \mathrm{CK})}{1 + \mathrm{BH} + \mathrm{I}(\mathrm{BC} + \mathrm{D})} = -744.6\ \mu\mathrm{V/V} \quad \textbf{(10.19)}$$

$$\mathrm{T_o} = \frac{v_L}{v_{c2}} = \mathrm{G} = 996.8\ \mathrm{mV/V} \quad \textbf{(10.20)}$$

The total gain can be found by using the following equation.

$$A_v = \frac{v_L}{v_S} = \frac{\mathrm{T_i\,T_g\,T_o}}{1 + \mathrm{T_g\,T_h}} = 20.79\ \mathrm{V/V} \quad \textbf{(10.21)}$$

The loop gain can be found by $\mathrm{T_g\ T_h}$, or -15.63 mV/V.

The simplest method for finding the input resistance is to use equation (5.6).

$$R_{in} = R_S\left(\frac{\mathrm{T_i}}{1 + \mathrm{T_g\,T_h} - \mathrm{T_i}}\right) = 7.605\ \mathrm{k\Omega} \quad \textbf{(10.22)}$$

The other approach to finding the input resistance involves using equation (5.4). To do this, we must first find the open-loop resistance. Caution must be taken when computing the open-loop input and output resistance because of intermediary feedback loops in the system. To find the input resistance, we begin by grounding the node v_{c2}. Looking at v_{c2}, we see a closed-loop shunt-feedback amplifier with Q_1. We can therefore apply equation (9.6) to find the open-loop input resistance of the feedback loop.

$$R_{in,ol} = R_{B1} \parallel \left[(\beta + 1)\left(r_{e1} + R_{E1} \parallel R_f\right)\right] \parallel \left(\frac{R_{f1} + R_{C1} \parallel R_{f2} \parallel R_{B2} \parallel r_{B2}}{\alpha \cdot \dfrac{R_{C1} \parallel R_{f2} \parallel R_{B2} \parallel r_{B2}}{r_{e1} + R_{E1} \parallel R_f} + 1}\right) \quad \textbf{(10.23)}$$

$$\rightarrow R_{in,ol} = 2.217\ \mathrm{k\Omega} \quad \textbf{(10.24)}$$

The next step is to divide the open-loop resistance by one plus the loop gain, or $1 + \mathrm{T_g\ T_h}$. Since we want to eliminate the effect of R_S on the input resistance, we must first recalculate the path gains H and I so that R_S is infinitely large. This in turn changes $\mathrm{T_h}$, so the overall loop gain of the system changes.

$$\mathrm{H}\big|_{R_S \to \infty} = R_{B1} \parallel R_{f1} = 9.091\ \mathrm{V/mA} \quad \textbf{(10.25)}$$

$$\mathrm{I}\big|_{R_S \to \infty} = -\frac{R_{B1}}{R_{f1} + R_{B1}} = -90.91\ \mathrm{mA/V} \quad \textbf{(10.26)}$$

$$\mathrm{T_h}\big|_{R_S \to \infty} = -\left.\frac{\mathrm{HK} + \mathrm{I}(\mathrm{J} + \mathrm{CK})}{1 + \mathrm{BH} + \mathrm{I}(\mathrm{BC} + \mathrm{D})}\right|_{R_S \to \infty} = -33.76\ \mathrm{mV/V} \quad \textbf{(10.27)}$$

The loop gain of the system with R_S removed is therefore -708.5 mV/V.

$$R_{in} = \frac{R_{in,ol}}{1 + \mathrm{T_g T_h}\big|_{R_S \to \infty}} = 7.605\ \mathrm{k\Omega} \tag{10.28}$$

Another way of finding the input resistance is by noticing in Figure 10.5 that the nodes v_{b1} and v_{c1} form a feedback loop. We can find the closed-loop input resistance of this loop by first finding the open-loop resistance by zeroing the node v_{c1}, then dividing that quantity by one plus the loop gain of the loop.

$$R_{in,ol}\big|_{v_{c1}=0} = R_{B1} \,||\, \left[(\beta+1)\left(r_{e1} + R_{E1} \,||\, R_f\right) \right] ||\, R_{f1} = 1.791\ \mathrm{k\Omega} \tag{10.29}$$

$$R_{in,ol} = \left. \frac{R_{B1} \,||\, \left[(\beta+1)\left(r_{e1} + R_{E1} \,||\, R_f\right) \right] ||\, R_{f1}}{1 + (\mathrm{BC} + \mathrm{D})\left(\dfrac{\mathrm{I}}{1 + \mathrm{BH}} \right)} \right|_{R_S \to \infty} = 2.217\ \mathrm{k\Omega} \tag{10.30}$$

Finally, the results in equation (10.31) are divided by 1 + $\mathrm{T_g}$ $\mathrm{T_h}$ to find the closed-loop input resistance. Similarly, when finding the output resistance, we are faced with a challenge of looking back into a complicated feedback loop. Because the K path is not cancelled after grounding the error node v_{b1}, the circuit topology does not exactly match the shunt-feedback amplifier in Figure 9.1, so equation (9.7) will not help us find the output resistance. However, we have already determined the loop gain of the shunt-feedback amplifier with Q_2, which is -E (J + C K). This is apparent in Figure 10.5. To begin, we will remove the emitter-follower stage from the circuit by letting i_{b3} be 0. The following path gains are affected by removing the output stage.

$$\mathrm{E}\big|_{i_{b3} \to 0} = \frac{R_{C2} \,||\, \left(R_f + R_{E1} \,||\, r_{e1}\right)}{R_{f2} + R_{C2} \,||\, \left(R_f + R_{E1} \,||\, r_{e1}\right)} - \alpha \cdot \frac{R_{L2} \,||\, R_{f2} \,||\, \left(R_f + R_{E1} \,||\, r_{e1}\right)}{r_{e2} + R_{E2}} \tag{10.32}$$

$$\to \mathrm{E}\big|_{i_{b3} \to 0} = -13.50\ \mathrm{V/V} \tag{10.33}$$

$$\mathrm{F}\big|_{i_{b3} \to 0} = \frac{R_{E1}}{r_{e1} + R_{E1}} \cdot \frac{R_{C2} \,||\, R_{f2}}{R_f + r_{e1} \,||\, R_{E1} + R_{C2} \,||\, R_{f2}} = 558.0\ \mathrm{mV/V} \tag{10.34}$$

$$\mathrm{T_g}\big|_{i_{b3} \to 0} = \left. \frac{\mathrm{E}(\mathrm{BC} + \mathrm{D}) + \mathrm{F}}{1 + \mathrm{E}(\mathrm{J} + \mathrm{CK})} \right|_{i_{b3} \to 0} = 21.00\ \mathrm{V/V} \tag{10.35}$$

The loop gain of the entire system with the emitter-follower stage removed, $\mathrm{T_g}$ $\mathrm{T_h}$, is therefore -15.63 mV/V, and the loop gain of the feedback amplifier with Q_2 is 16.35 V/V. The open-loop resistance looking into the node v_{c2} with the emitter follower stage removed is therefore given in equation (10.36). We then find the closed-loop output resistance by dividing by the loop gain $\mathrm{T_g}$ $\mathrm{T_h}$.

$$R_{v_{c2},ol}\big|_{i_{b3} \to 0} = \left. \frac{R_{C2} \,||\, \left(R_f + R_{E1} \,||\, r_{e1}\right) ||\, R_{f2}}{1 + \mathrm{E}(\mathrm{J} + \mathrm{CK})} \right|_{i_{b3} \to 0} = 103.2\ \Omega \tag{10.36}$$

$$R_{v_{c2},cl}\Big|_{i_{b3}\to 0} = \left.\frac{R_{C2} \,||\, \left(R_f + R_{E1} \,||\, r_{e1}\right) \,||\, R_{f2}}{\left[1+\mathrm{E}\left(\mathrm{J}+\mathrm{C}\,\mathrm{K}\right)\right]\left(1+\mathrm{T_g}\,\mathrm{T_h}\right)}\right|_{i_{b3}\to 0} = 104.9\ \Omega \quad \textbf{(10.37)}$$

Finally, we calculate the output resistance looking into the emitter of the common collector stage after first removing R_L.

$$R_{out} = R_{E3} \,\Bigg\|\, \left\{ r_{e3} + \left.\frac{R_{C2} \,||\, \left(R_f + R_{E1} \,||\, r_{e1}\right) \,||\, R_{f2}}{\left(\beta+1\right)\left[1+\mathrm{E}\left(\mathrm{J}+\mathrm{C}\,\mathrm{K}\right)\right]\left(1+\mathrm{T_g}\,\mathrm{T_h}\right)}\right|_{i_{b3}\to 0} \right\} = 11.22\ \Omega \quad \textbf{(10.38)}$$

11. Conclusion

In this paper, we discussed various techniques for analyzing feedback circuits. We saw how feedback theory can apply for simple circuits including only resistive elements as well as more complicated ones with discrete BJT transistors and multiple feedback paths. We saw what effects the loop gain had on the input and output resistance of circuits. The core concepts discussed in this paper can be carried further, and be used to evaluate the stability and bandwidth of circuits with reactive components such as inductors and capacitors.

12. References

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